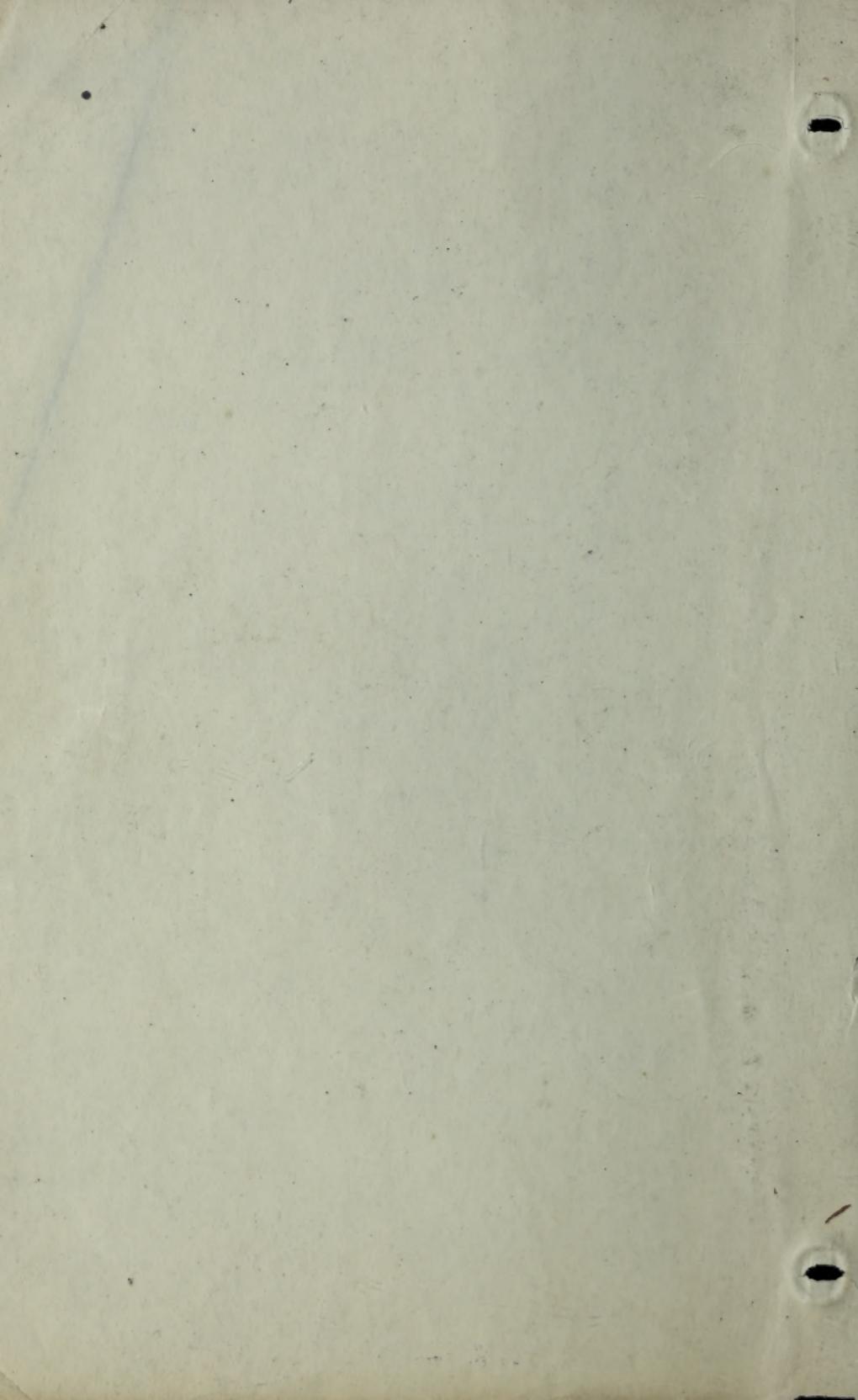


625.11
H69h

~~Eugene~~
~~EL King~~

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THE
HOLBR DOK
SPIRAL.



625.11

H69 HOLBROOK'S SPIRAL.

If R be the radius of curvature at any point and L the distance of this point from the tangent point measured along the curve, and A a constant, then the definition of this curve is expressed

$$(1) RL = A$$

The following calculations are for a spiral whose degree increases one minute per foot - then

$$(2) RL = A = \frac{100 \times 60000}{3.14159} = 343770$$

(3) $R = \frac{343770}{L}$ Column headed R calculated from (3). Also, it follows that for the inclination of the curve at any point to the tangent we have

$$(4) \Delta = \frac{L^2}{R^2} = 0.0000014545 L^2 \text{ and representing } 0.0000014545 \text{ by } A, \text{ we have}$$

$$(5) \Delta = A L^2 \text{ In minutes (5) becomes}$$

$$(6) \Delta = .005 L^2 \text{ Column headed } \Delta, \text{ calculated from (6).}$$

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Let the curve be referred to rectangular axes with origin at P.C. and the tangent as axes of Y.

For convenience in laying out let us find X and Y in terms of L. To obtain this we have

$$(7) \frac{dy}{dL} = \cos. \Delta_i = \cos. A_i L^2 \text{ hence } Y = \int_0^L \cos. A_i L^2 dL = L - \frac{120^2 L^5}{1.2.3.4.5} + \frac{16800^4 L^9}{1.2.3.4.5.6.7.8.9} - 8c.$$

Reducing we have:

$$(8) Y = L - 0.00000000000201557. L^5 + 8c.$$

$$(9) \frac{dx}{dL} = \sin. \Delta_i = \sin. A_i L^2 \text{ hence } X = \int_0^L \sin. A_i L^2 dL = \frac{20. L^3}{1.2.3} - \frac{120. 0^3 L^7}{1.2.3.4.5.6.7} + 8c. \text{ Redue-}$$

ing we have

$$(10) X = 0.0000004848 L^3 + 8c.$$

X and Y in the tables are calculated from (8) and (10).

If x_0, y_0 be the coordinates of the point where the circle joining the spiral to given point x, y ,

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becomes parallel to the tangent we have.

$$(11) X_0 = X - R \cos \sin D.$$

(12) $Y_0 = Y - R \sin D$. Columns headed X_0 and Y_0 are calculated from (11) and (12). If D be the deflection necessary to be turned off from any point Y , on the tangent, tangent to any point X , on the curve we have

$$(13) \tan D = \frac{X}{Y - y}.$$

If we choose P.C. as a point from which to turn off deflections $Y=0$ and (13) becomes

(14) $\tan D = \frac{X}{Y}$. Column headed D is calculated from (14). D , is calculated for $Y=200$.

Column C gives the length of chord corresponding to arc of 50 ft. for various degrees of curvature.

Let Δ = intersection angle, T = length of tangent and T_c = length of tangent of circular curve of some radius as middle part of

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curve and having intersection

$= \Delta$ or $T = R \ Tan \frac{1}{2} \Delta$ Then we have

(15) $T = T_0 + X_0 \ Tan \frac{1}{2} \Delta + Y_0$ Substituting

(16) $T = (R + X_0) \ Tan \frac{1}{2} \Delta + Y_0$

Having found the length of the tangents by (16) and fixed the tangent points on the ground the spiral ends may be run in by the deflection method using values taken from columns headed D or D' , according as the transit is at P.C. or at 200ft. from it on the tangent towards the apex, or points may be located by ordinates found under head of X , or by any combination of these methods desired.

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FIRST PROBLEM.

Given: Two tangents joined by a simple curve; required, the intersection angle being $27^{\circ}15'$, to put in a 70° curve with spirals having a ratio of increase of 1° in $60'$.

$$\text{By (16)} \quad t = (r + x_0) \tan \frac{t}{r} + y_0 = 554.99 \text{ ft.}$$

Having established the tangent points A-A', figure 2 was set over each end tangent from the tangents the deflections found in the table under D. The instrument for the last deflection should read $0^{\circ}58'18''$. The sum of the deflections for the circular arc

$$= \frac{1}{2} \Delta - \Delta_1 = 13^{\circ}37'30'' - 2^{\circ}42'00'' = 10^{\circ}55'30''$$

And the length of the circular arc

$$= \frac{10^{\circ}55'30''}{1^{\circ}30'} \times 100 = 720.333 \text{ ft.}$$

Required: The intersection angle being $70^{\circ}30'$, to put in a 70° curve, with spirals having a ratio of increase of 1° in 50 ft. The sum of the deflections for the circular arc

$$= \frac{1}{2} \Delta - \Delta_1 = 34^{\circ}15'$$

And the length of the circular arc

$$= \frac{34^{\circ}15'}{1^{\circ}00'} \times 100 = 342.5 \text{ ft.}$$

The back sight given, 100', being too short for accurately running in so long a curve, by the proceeding method, we may proceed as follows:

Establish the points E-E', figure 2, by $BD = R$ $\text{Tan } \frac{1}{2} \Delta = 2022.7 \text{ ft.}$ and $BE = \frac{1}{2} R = 9.146 \text{ ft.}$

As long a back sight as is desired can then be obtained by measuring off from a point down the tangent the distance 9.146 ft. The circular curve is then run in from the auxiliary tangents, the points F-F' being fixed by deflecting for the chord

$$y_0 = 50 \text{ ft.}$$

The points A-A' being got by measuring back from B-B' the distance

$$y_0 = 50 \text{ ft.}$$

The instrument is set up at A or A', and the spiral run in by deflections as before. Should it happen, as is sometimes the case in sharp curves, that the entire spiral is not visible from A or A', the instrument may be moved two hundred feet along the tangent toward the apex, and further points established by turning off from the tangent deflections found in the table under d.

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SECOND PROBLEM.

Given: Two tangents joined by a simple curve, to find a circular arc with spirals joining the same tangents, that will replace the simple curve on the same ground as nearly as may be.

It is evident that the new circular arc must be outside the old curve at the middle point H , since the application of spirals draws the curve inward, the distance O_1O_2 ; and that to retain the same tangents, the radius of the new curve must be less than that of the old.

Let FE be the curve to be replaced with a new curve and spirals.

If $DE = x_0$, and $SC = \text{the Spiral}$, then will SCD be the old curve with Spirals.

An inspection of figure 3 will show that SCD is inside the curve for its entire length. By trigonometry

EV-ResectA.

Now if we can find a curve with a radius R' , such that R' except λ will be such a quantity, that when increased by x_0 the curve will still be outside the point E , we will have a curve which will nearly coincide with SFE , a little outside of E and a little inside of F . It is evident that R' is always shorter than R and x_0 , for R' is a little longer than x_0 for R (see tables), & the x_0 for R' being always a little larger than x_0 for R , the following formula may therefore be used in finding R' .

$$R = \frac{\text{Actual } R - 1.2 \times 0}{\sec \theta} \dots \dots \dots (17)$$

Required: The intersection angle being $30^{\circ}00'$, to replace a 6° curve with a new curve, with spirals having a rate of increase of 1° in 30 , and replacing the simple curve on the same ground as nearly as may be.

$$\begin{aligned} \text{By (17) } R' &= \frac{R \tan \sec \frac{\pi}{12} \lambda - 1\% x_0}{\sec \frac{\pi}{12} \lambda} \\ &= \frac{(954.93 \times .035276) - 2.11}{.035276} \\ &= 897.95 \text{ ft.} = R \text{ for a } 6^\circ 23' \text{ curve} \end{aligned}$$

Deciding, therefore, upon the use of a 6°20'; we have

$$By (16) t = (R + x_0) \tan \frac{\theta}{2} \Delta y_0 = 337.0 \text{ m}$$

A and A' are established, and the ordinary curve runs as in the first problem. The radii in the tables commonly in use, are calculated for chord lengths instead of arc lengths. The radii of these tables should therefore be always used, and when spirals are used on curves with very short radii, the deflections on the circular arc can be made for short chords of fifty, or twenty-five feet, or the chord length calculated.

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THIRD PROBLEM.

Given: Two tangents joined by a simple curve, to find a circular arc having one spiral on each end, joining the same tangents. Because of the end of the curve being on a bridge, or other special feature in its location, it is sometimes desirable to retain one end of the new curve more nearly in its old position than is possible with the application of spirals to both ends. The end of the curve may then become a part of the circular arc, and a spiral applied to the free end only. The point A, figure 4, may be found by the formula:

$$t' = R \tan. \frac{1}{2} \lambda + x_0 (\sec 90^\circ - \lambda) \dots\dots\dots (18)$$

The point A of the spiral may be found by the formula:

$$t'' = R \tan. \frac{1}{2} \lambda + y_0 - x_0 (\tan. 90^\circ - \lambda) \dots\dots\dots (19)$$

Required: The intersection angle being $62^\circ 26'$, to replace a $90^\circ 15'$ by a new curve, with a spiral having a rate of increase of one degree in twenty feet on one end only.

$$\begin{aligned} \text{By (18)} \quad t' &= R \tan. \frac{1}{2} \lambda + x_0 (\sec 90^\circ - \lambda) \\ &= 619.41 X .60602 + 2.32 X 1.12807 \\ &= 377.93 ft. \end{aligned}$$

$$\begin{aligned} \text{By (19)} \quad t'' &= R \tan. \frac{1}{2} \lambda + y_0 - x_0 (\tan. 90^\circ - \lambda) \\ &= 619.41 X .60602 + 22.93 - 1.21 \\ &= 466.59 ft. \end{aligned}$$

Having established A and A', the new curve with spiral may then be run in as shown under the first problem.

FOURTH PROBLEM

Given: Two tangents joined by a simple curve, to find a circular arc having on each end spirals differing in their rates of increase, joining the same tangents.

From an inspection of figure 4 it will be seen that

$$t' = R \tan. \frac{1}{2} \lambda + y_0 + x_0 (\sec 90^\circ - \lambda) - x_0 (\tan. 90^\circ - \lambda) \dots (20)$$

$$t'' = R \tan. \frac{1}{2} \lambda + y_0' + x_0 (\sec 90^\circ - \lambda) - x_0' (\tan. 90^\circ - \lambda) \dots (21)$$

Required: The intersection angle being $37^\circ 40'$, to replace a five degree curve by a new curve, having on one end a spiral increasing one degree in sixty feet, and on the other end a spiral increasing one degree in thirty feet.

For the end having a spiral increasing

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1° in 60 ft.

$$\text{By (20) } t^2 = (1145.9 \times .34108) + 198.92 + (82 \times 1.63648) - \\ (3.29 \times 1.29541) \\ = 537.89 \text{ ft.}$$

For the end having a spiral increasing 1° in 30 ft.

$$\text{By (21) } t^2 = (1145.9 \times .34108) + 25.00 + (329 \times 1.63648) - \\ (82 \times 1.29541) \\ = 470.16 \text{ ft.}$$

From the points thus found, run in the spirals and connect them with a fine degree curve, using deflections for fifty feet chords as in the first problem.

FIFTH PROBLEM.

To connect two tangents by a reverse curve with spirals.

In the treatment of reverses curves, a formula which will enable us to determine the radius from a fixed length of tangent, instead of determining the tangent from a given radius, will be very convenient. We may determine such equations as follows. It will be seen from the tables that

$2Y_0 = Y$ very nearly, hence $Y_0 = R \sin. \lambda$ nearly.

The tables also show that x_0 is always small when compared with R , and may be discarded. (16) then becomes

$$R \tan. \frac{\lambda}{2} \approx t - R \sin. \lambda \\ \text{or } \tan. \frac{\lambda}{2} \approx \frac{t}{R} - \text{aa}, t^2 \text{ nearly}$$

$$\text{or } \frac{1}{a} \tan. \frac{\lambda}{2} \approx \frac{t}{R} - t^2 \text{ nearly}$$

$$\text{but aa} = \frac{t}{R} [\text{see 1 and 4}]$$

$$\text{hence } t^2 - 2tt = -\frac{t}{a} \tan. \frac{\lambda}{2} \text{ nearly}$$

$$\text{and } t = t - \sqrt{t^2 - \frac{t}{a} \tan. \frac{\lambda}{2}} \dots \dots \dots \quad (22)$$

A little study of figure five will show that the spirals joining the two branches must be carried to H , where the radii will be infinite, and this is a common point viz.: the point of reversal of curvature for two circular arcs joining the given tangents; find, therefore the point H , and the other two points, as for plain circular curves. Having determined these points, the radius, length of spirals, etc. may be found from (22). The two parts of the curve when thus located will be found to be considerably outside the two

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5

circular arcs, joining the two tangents. The method does very well for new work, but no method of applying a segment approaches so far back devised which will locate new curves with spirals nearly enough on the ground occupied by the old curves (if the old curves have short radii), to be of any practical use in maintenance of way work, because the application of spiral requires a lengthening of the tangents equal to nearly half the length of the spirals, and in reverse curves the length of the common tangents is fixed.

Required: The intersection angles being 40° and 60° , the common tangent, ~~one hundred feet~~, there being used with one arm five hundred feet, with the other four hundred feet. The curve having tangents of five hundred feet to be fitted with spirals increasing one degree in thirty feet, and the curve having tangents of four hundred feet to be fitted with spirals increasing one degree in ten feet.

$$\text{By (22)} \quad l = 300 - \sqrt{250000 - 343775x} - 36397$$

$$l' = 147 \text{ ft. nearly}$$

$$l' = 400 - \sqrt{160000 - 114600x} - 57735$$

$$= 94 \text{ ft. nearly}$$

The tables show, by interpolation, that the radius for the first arm is 47.946 ft. being that of a $90^\circ 24'$ curve and that the radius for the second arm is 61.45 ft. being that of a $90^\circ 24'$ curve. The length of the spirals and the radii of the corresponding arcs having been so determined they may be run in as in the first problem.

SIXTH PROBLEM

Given: Two tangents joined by a compound curve, to replace the same with two simple curves, joined to the tangents and to each other by spirals.

It is obvious that the joining of the two simple curves to the tangents presents no difficulties. Selecting for this purpose the branch with the shortest radius, its beginning point may be determined by the method given in the third problem, adding to the tangent of the circular curve the length:

$$Y - X_0 \text{ Tan.}(90^\circ - A') \dots \dots \dots (23)$$

From an examination of figure seven, which shows in detail that part of figure six between R and S, it will appear that the con-

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men tangent to the two circular curves
their compounding point will be moved
toward the apex the distance $CD = P$, which
is found from

$$T_{\text{eff}}, j = \frac{Y_0 - Y_0'}{(R'' - R') - (X_0' - X_0'')} \quad \dots \dots \dots \quad (24)$$

$$\text{and that } \rho = R'' R' - \frac{y'_0 - y''_0}{\sin j} \dots \dots \dots \quad (25)$$

This quantity p so nearly equals x , for the length of the spiral joining the two circular pts, that the latter may conveniently be substituted for it.

The length of the tangent of the branch with the longer radius will therefore be increased by

$$y''_o - x''_o \operatorname{Tan}(90^\circ - \hat{\alpha}) = \rho \sec(90^\circ - \hat{\alpha}) \quad \dots \dots \dots (2.6)$$

Observing now the new positions assumed by the circular arcs, they will be seen to overlap each other by a distance equal to

$$x'_0 \sec(90^\circ - \gamma) + x''_0 \sec(90^\circ - \gamma) + y \tan(90^\circ - \gamma) = \dots \quad (27)$$

Measuring back along the first branch
from its end a distance

$$x_0 \sec(90^\circ - \gamma) \dots \dots \dots \quad (28)$$

A point will be found opposite the middle point of the spiral connecting the two branches;

The length of the connecting spiral will be equal to as many feet as there are minutes in the difference in degreee of the two branches of the curve, if a spiral increasing one degree in sixty feet be used; two-thirds, if one increasing one degree in forty feet; one-half, if one increasing one degree in thirty-feet, etc.

The length of the connecting spiral and the position of a point opposite its center having been thus determined, its two ends may be fixed by measuring off on each branch from the determined point one-half its length. Having thus established points, the spiral connecting them may be run in as follows:

in as follows: From A turn off from the tangent to the circular arc at that point the angle $90^\circ - \gamma$ as shown in figure 56, and lay off there corresponding to A from S turn off from the tangent to the circular arc at that point the angle $90^\circ - \gamma$ and lay off a corresponding to H . The two

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points found are on the tangent from which the connecting spiral springs. Its origin will be distantly from A' from the point located from S. From this last point run in the spiral connecting the two arcs from S to A, as in the first problem.

It will be seen that this solution permits of the use of spirals having different rates of increase for each end of the compound curve, and for the connecting spiral at the compounding point, and that it may also be applied to compound curves having more than one compounding point.

Required: The intersection angle between two tangents being $65^{\circ} 00'$ to connect the same by a compound curve—the one branch a five degree curve, intersection angle $50^{\circ} 00'$; the other branch a three degree curve, intersection angle $15^{\circ} 00'$ —with spirals joining the tangents and connecting the circular arcs.

The tangent on the branch having the shorter radius is 700.79 feet. Using a spiral increasing one degree in forty feet, lengthens this by

$$(23) \begin{aligned} &= y_0 - x_0' \ Tan.(90^{\circ} - \lambda') \\ &= 99.98 - (1.45 \times 8.3910) \\ &= 98.76 \end{aligned}$$

The tangent on the branch having the longer radius is 915.59 feet. Using a spiral increasing one degree in fifty feet, and for the connecting spiral one increasing one degree in fifty feet, lengthens this by (26)

$$(24) \begin{aligned} &\tan j = \frac{y_0 - y''_0}{(r'' - r') - (x_0' - x''_0)} \\ &= \frac{99.98 - 90.00}{(1909.85 - 1145.91) - (1.45 - .707)} \\ &= 0.01307 \\ &= 0^{\circ} 45' \end{aligned}$$

$$(25) \rho = r'' - r' - \frac{y_0 - y''_0}{\sin j} \\ = 1909.85 - 1145.91 - \frac{99.98 - 90.00}{0.01309} \\ = 153$$

$$(26) \begin{aligned} &= y''_0 - x''_0 \ Tan.(90^{\circ} - \lambda'') - \rho \ sec(90^{\circ} - \lambda'') \\ &= 90.00 - (.707 \times 3.73205) - (.153 \times 3.86370) \\ &= 86.77 \end{aligned}$$

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If 05 is suggested, p be taken as equal in value to the $\frac{1}{2}$ of the length of the connecting spiral, then would

$$(26) = y_0'' - x_0'' \tan(90^\circ - \lambda') - p \sec(90^\circ - \lambda'')$$
$$= 90.00 - (.707 \times 3.73205) - (.146 \times 3.86370)$$
$$= 86.80$$

A difference of three-hundredths of a foot, about the width of a rock head. As this is much closer than the actual results to be obtained from the field work, the calculation of (24) and (25) may well be omitted.

Having fixed the points on the tangents where the branches with spirals begin, run them in for their full length. The ends will lie on parallel tangents separated by 146 feet, and will be $\frac{1}{2}$ of the distance

$$(27) = x_0' \sec(90^\circ - \lambda') + x_0'' \sec(90^\circ - \lambda'') + p \tan(90^\circ - \lambda'')$$
$$= (1.45 \times 1.90541) + (.707 \times 3.86370) + (.146 \times 3.73205)$$
$$= 5.17$$

Measuring back from the end of the curve having the shorter radius a distance

$$(28) = x_0' \sec(90^\circ - \lambda')$$
$$= (.146 \times 1.90541)$$
$$= 1.98$$

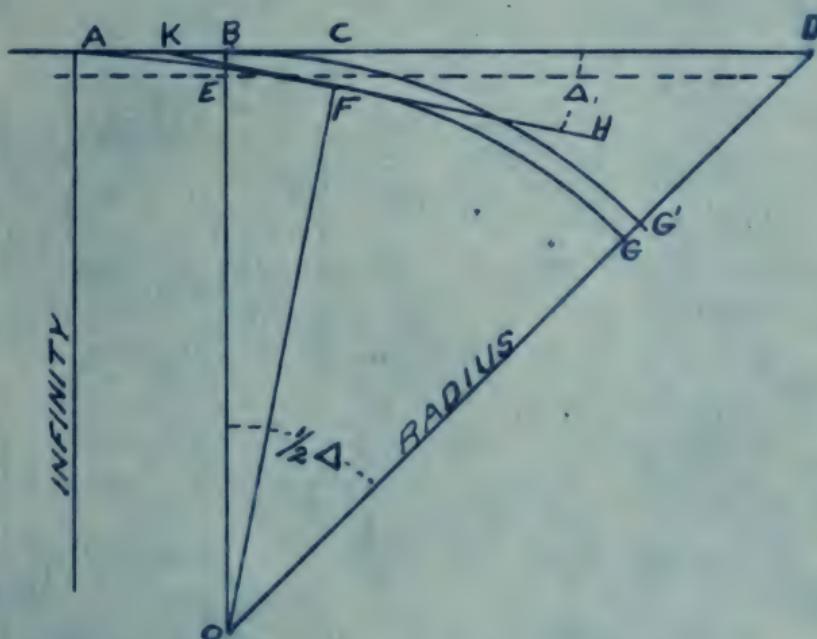
Fixes the point opposite the center of the connecting spiral, and measuring half the length of the connecting spiral (fifty feet), in both directions on the branches, establish the points R and S.

Setting up now at R, turn off from the tangent to the circular arc at that point the angle $90^\circ - \lambda' = 83^\circ 45'$, and measuring along this line from R the distance $x' = 2.09$ ft. for radius = 1195.91.

Setting up then at S, turn off from the tangent to the circular arc at that point the angle $90^\circ - \lambda'' = 87^\circ 45'$, and measure along this line $x'' = 1.964$ ft. for radius = 1190.85. Two points found are in the line from which springs the spiral. Prolonging, therefore, this line, and measuring along it the distance $y'' = 147.954$, we obtain the origin of the connecting spiral, from which point the sonic may be run in, retaining for use, of course, only so much of it as lies between S and R.

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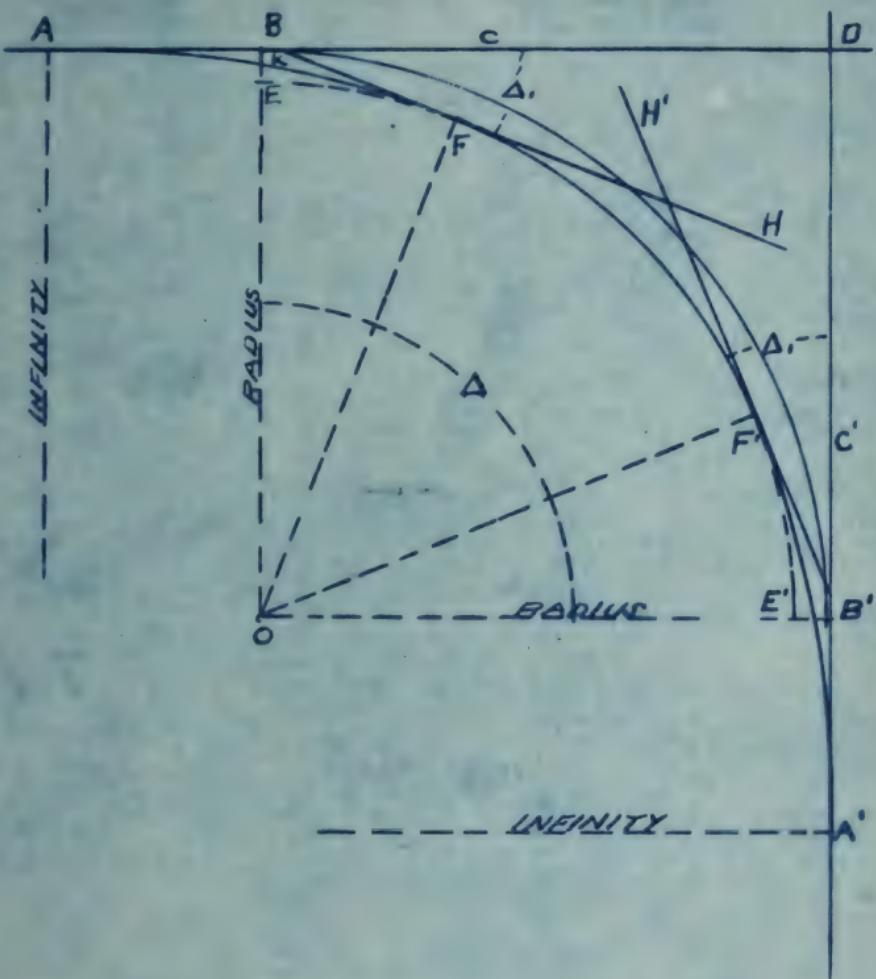
Figure 1.



BG' = original curve
 AFG = spiral & arc
 AD = tangent
 AC = Y
 CF = X
 AB = Y.
 BE = X.
 BOG = $\frac{1}{2}\Delta$
 DKH = Δ ,

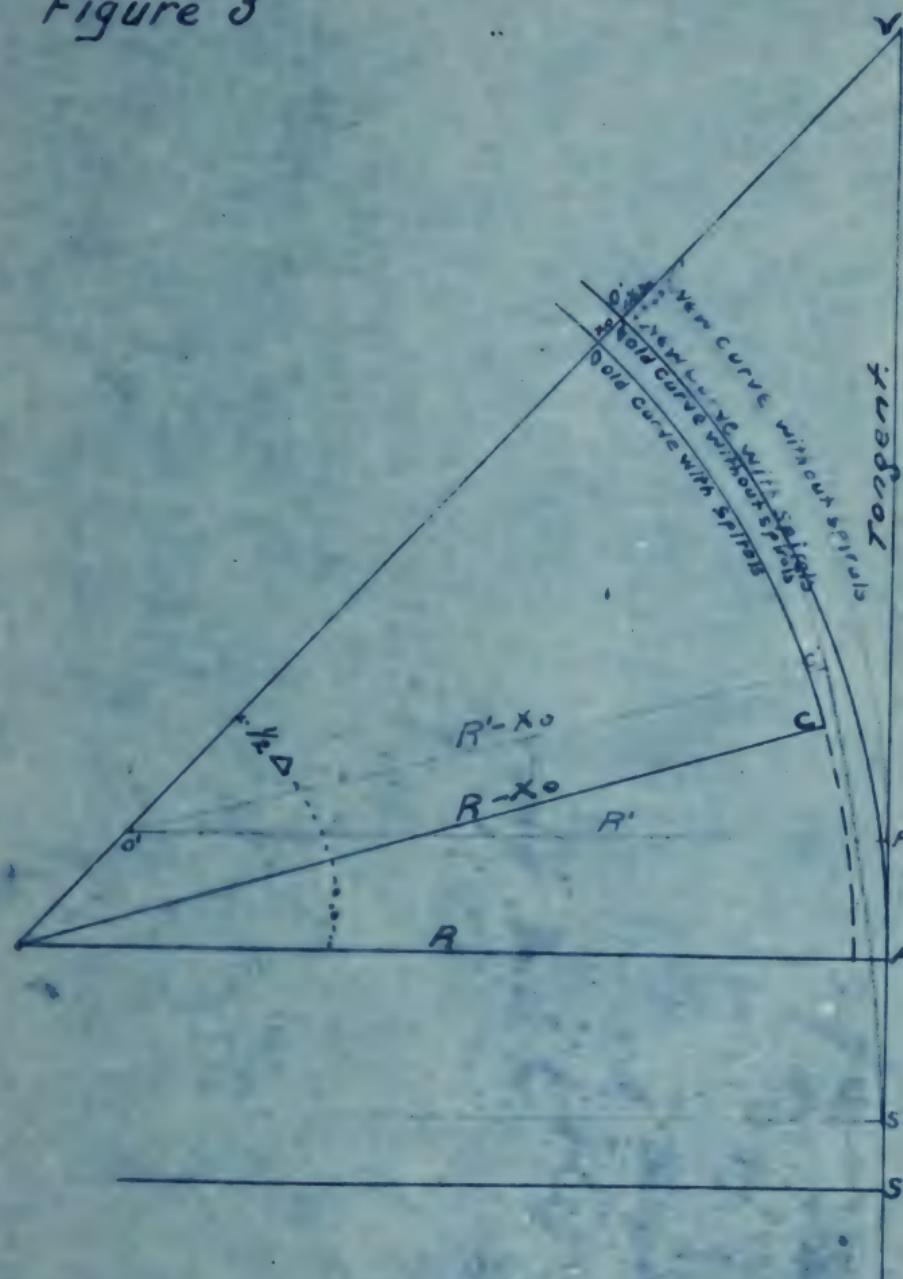
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Figure 2.



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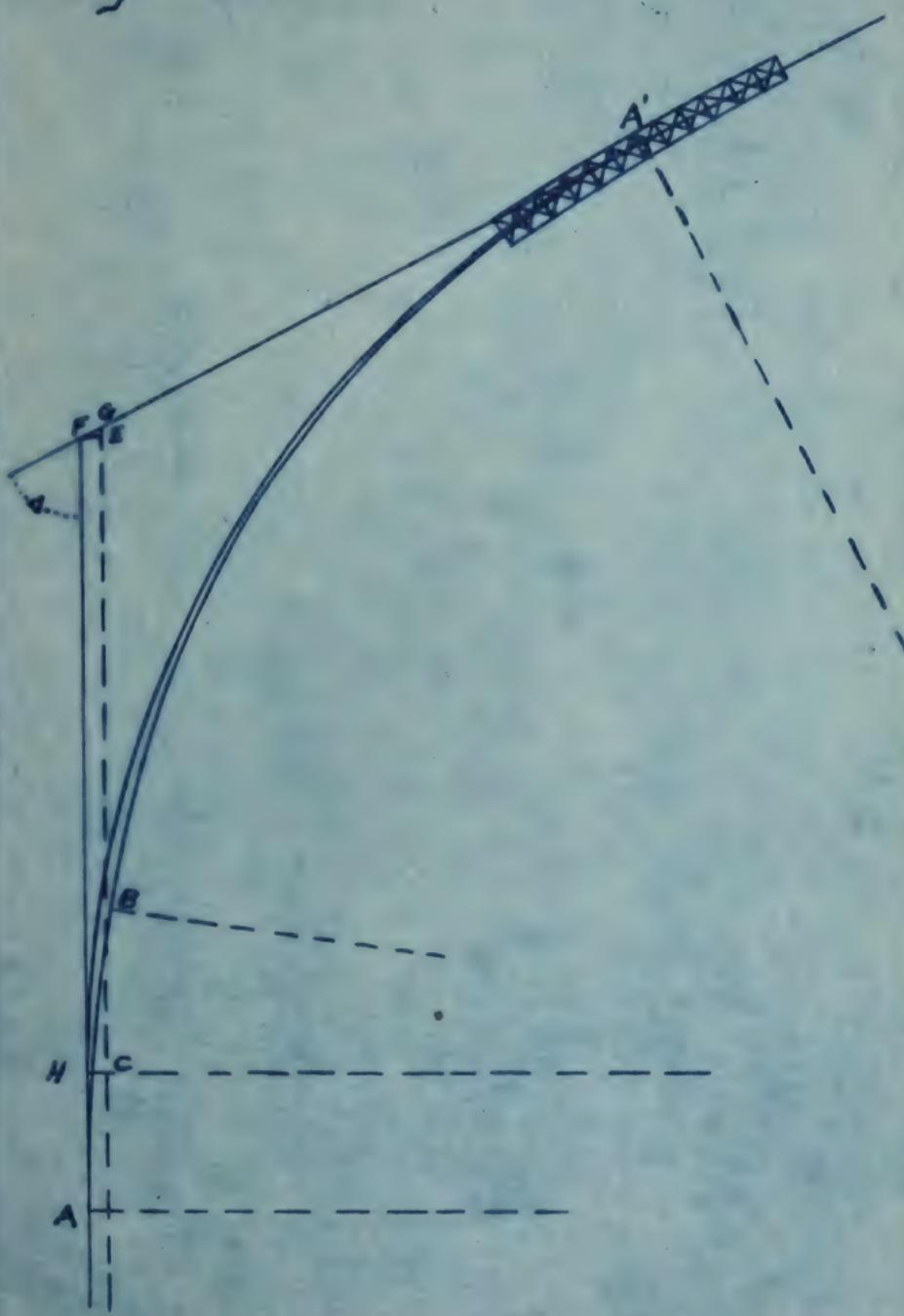
Figure 3



NOTE—WHITE LINES & LETTERS INDICATE FUNCTIONS OF OLD CURVE.
—RED LINES & LETTERS INDICATE FUNCTIONS OF NEW CURVE.

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Figure 4.



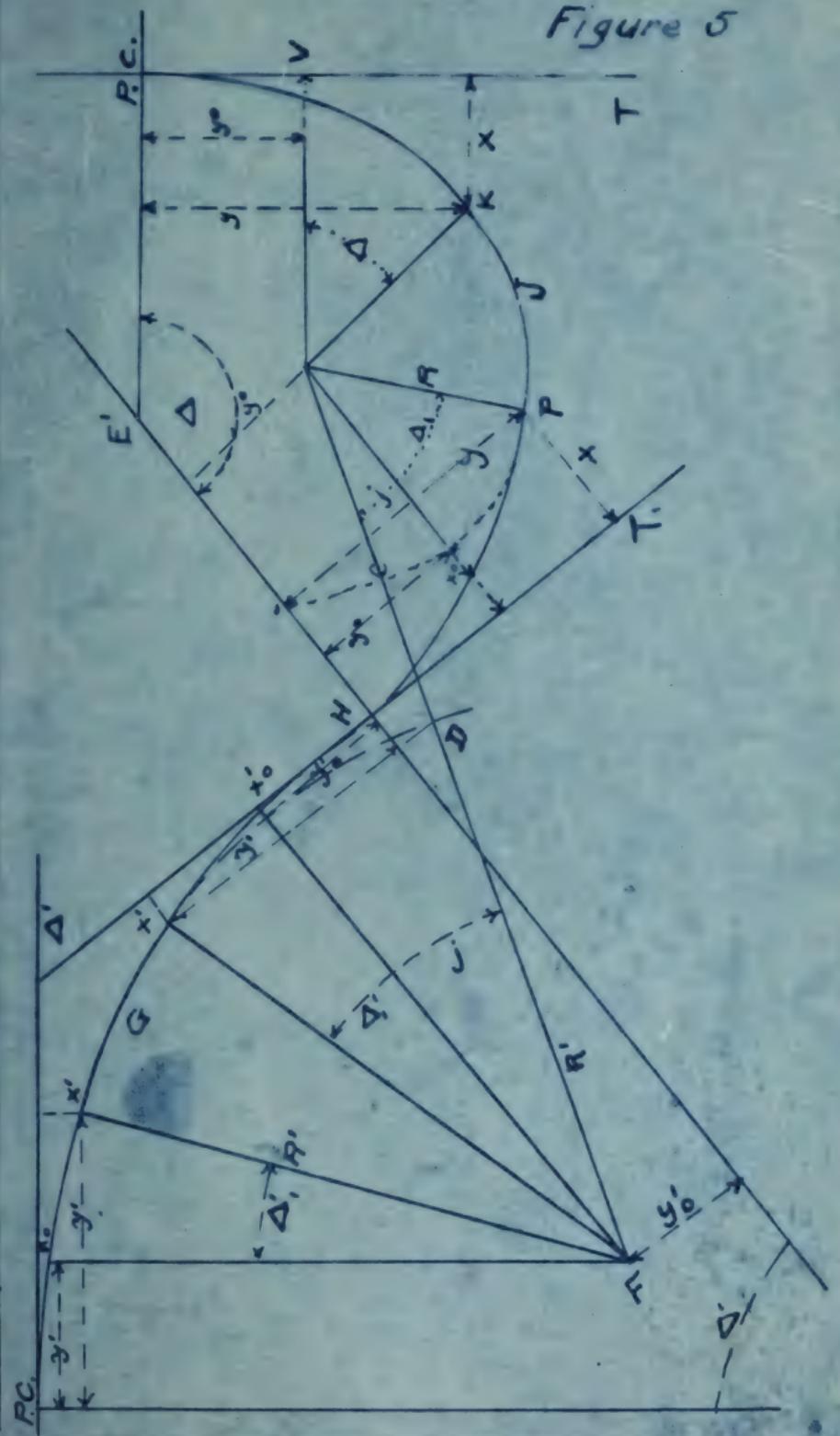
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Figure 4½



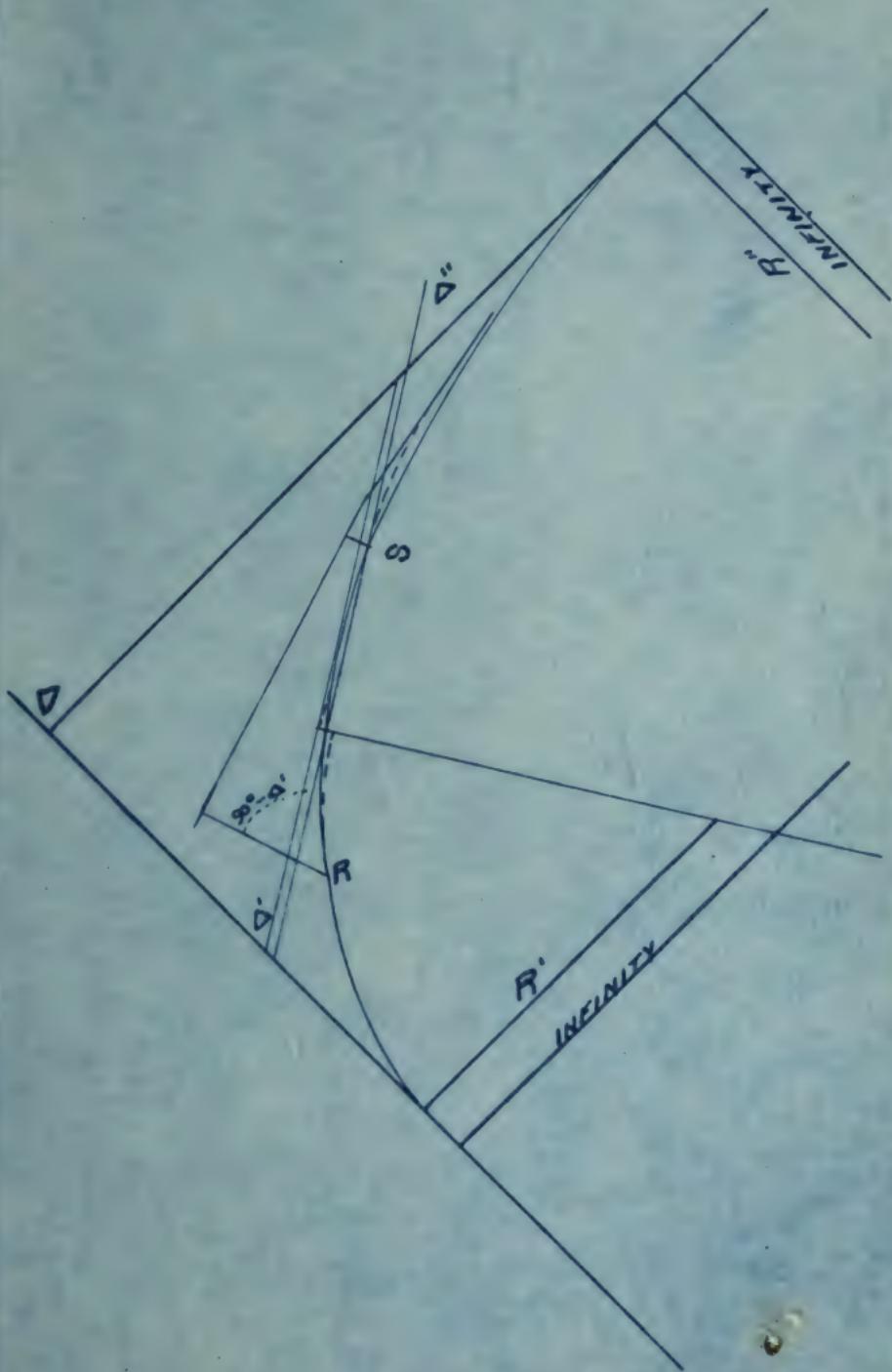
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Figure 5



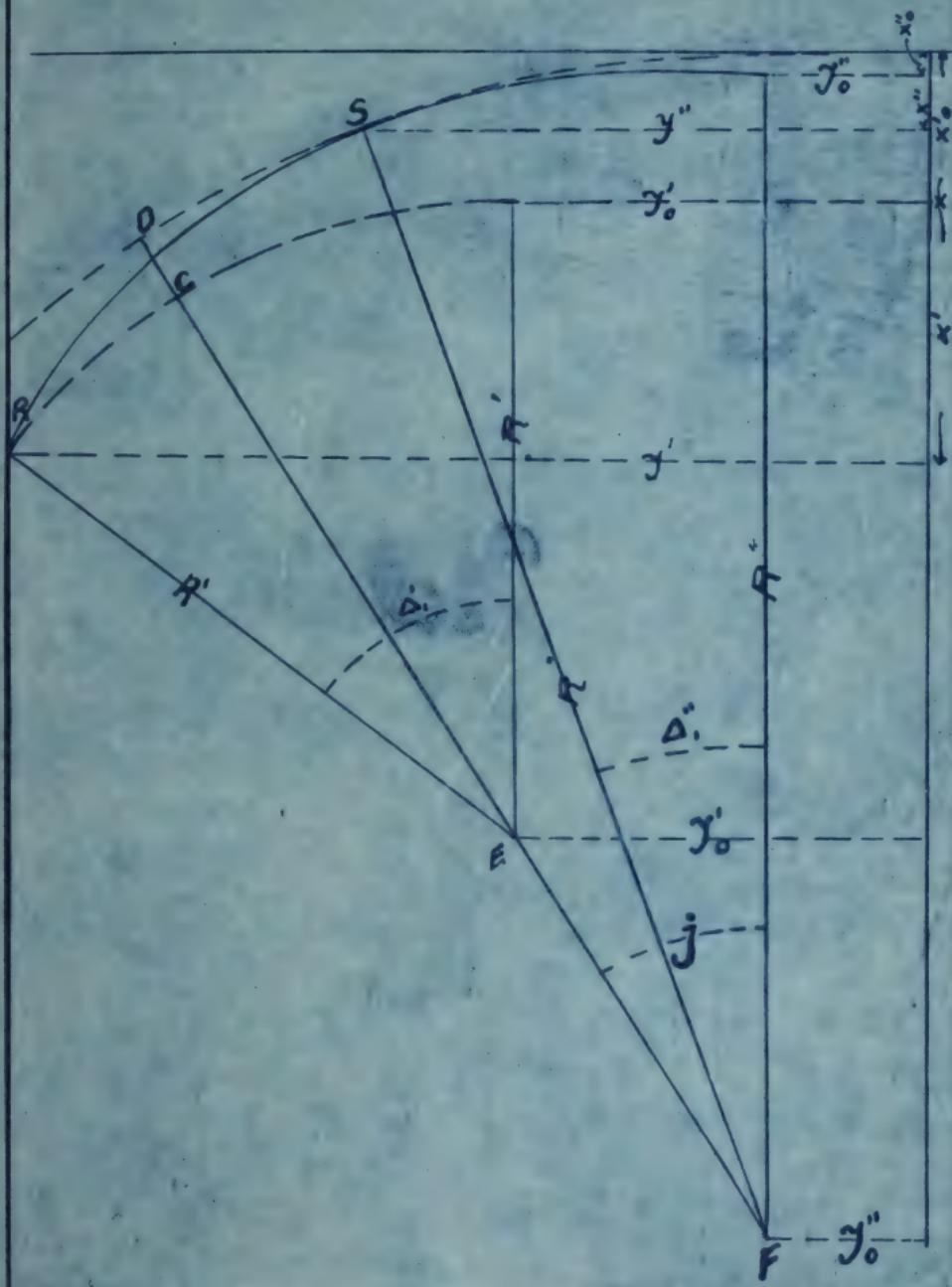
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Figure 6.



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Figure 7.



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Table of Spirals. Rate of Increase, $10^{\circ}/10'$

\angle	Deg.	Radius.	$\Delta.$	X.	Y	X_0	Y_0	D	D.
0	0° 00'	Infinity.	0° 00' 00"	.000	0.000	.000	0.000	00'	00'
10	1° 00'	3729.58	03'	.003	10.000	.001	5.000	01'	01'
20	2° 00'	2864.79	12'	.023	20.000	.006	10.000	03'	06'
30	3° 00'	1909.86	27'	.079	30.000	.021	15.000	09'	02'
40	4° 00'	1432.39	48'	.186	39.999	.046	20.000	16'	08'
50	5° 00'	1145.91	1° 15'	.364	49.998	.091	25.000	25'	02'
60	6° 00'	954.93	1° 48'	.628	59.994	.157	29.999	36'	00'
70	7° 00'	818.51	2° 27'	.998	69.987	.239	34.998	49'	02'
80	8° 00'	716.19	3° 03'	1.489	79.975	.372	39.996	04'	00'
90	9° 00'	636.62	4° 00'	2.121	89.953	.537	44.992	1°	02'
100	10° 00'	572.95	5° 00'	2.909	99.924	.724	49.988	40'	02'
110	11° 00'	520.87	6° 00'	3.872	109.977	.971	54.979	2°	01'
120	12° 00'	477.46	7° 12'	5.027	119.911	1.263	59.964	2°	24'
130	13° 00'	440.73	8° 27'	6.391	129.717	1.607	64.933	2°	49'
140	14° 00'	409.25	9° 48'	7.982	139.570	2.011	69.932	3°	20''
150	15° 00'	381.93	11° 15'	9.817	149.422	2.477	74.901	3°	34''
160	16° 00'	358.10	12° 48'	11.915	159.202	3.016	79.865	4°	16''
170	17° 00'	337.04	14° 27'	14.291	169.889	3.629	84.816	4°	48''
180	18° 00'	318.31	16° 12'	16.964	178.561	4.325	89.755	5°	10''
190	19° 00'	301.51	18° 03'	19.952	188.107	5.114	94.675	6°	03'
200	20° 00'	286.48	20° 00'	23.271	197.563	5.995	99.581	6°	43'
210	21° 00'	272.84	22° 03'	26.939	206.890	6.983	104.462	7°	25'
220	22° 00'	260.44	24° 12'	30.974	216.076	8.087	109.316	8°	09'
230	23° 00'	249.11	26° 27'	35.392	224.999	9.317	114.101	8°	26'
240	24° 00'	238.73	28° 48'	40.212	233.837	10.683	118.827	9°	54'

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Table of Spirals. Rate of Increase / in 10'

L	deg	Radius	Δ	X	Y	X.	Y.	D	D.
250	25° 00'	229.48	31°	45.45	242.57	12.097	23672/0°	36°	46° 52' 46"
260	26° 00'	220.368	33°	48°	250.87	18.733	28.287/1°	29°	40° 45' 06' 24"
270	27° 00'	212.206	36°	27°	257.255	259.073	15.543	27°	44° 44' 06' 16"
280	28° 00'	204.628	39°	12°	638.56	266.894	17.803	27°	18° 43° 40' 09"
290	29° 00'	197.571	42°	63°	70.944	274.381	20.002	29°	50° 43° 48' 44"
300	30° 00'	190.986	45°	00°	78.539	281.495	22.579	35°	22° 43° 56' 32"
310	31° 00'	184.825	48°	03°	86.658	288.149	25.386	44°	18° 44° 32' 04"
320	32° 00'	179.049	51°	12°	95.318	294.448	28.462	54°	18° 45° 16' 14"
330	33° 00'	173.624	54°	27°	104.536	300.198	31.884	56°	18° 45° 16' 14"
340	34° 00'	168.517	57°	48°	114.330	305.401	35.640	56°	18° 46° 13° 17°
350	35° 00'	163.702	61°	15°	124.718	310.005	39.755	56°	19° 47° 19° 39°
360	36° 00'	159.155	64°	48°	135.716	313.955	44.316	56°	19° 48° 25° 34"
370	37° 00'	154.853	68°	27°	147.343	317.18	49.370	57°	22° 42° 49° 58° 50°
380	38° 00'	150.778	72°	12°	159.616	319.663	54.930	57°	22° 42° 49° 58° 50°
390	39° 00'	146.912	76°	09°	172.551	321.295	61.056	57°	22° 42° 49° 58° 50°
400	40° 00'	143.239	80°	00°	186.680	322.022	67.807	57°	22° 42° 49° 58° 50°
410	41° 00'	139.654	84°	03°	200.482	321.776	75.306	57°	22° 42° 49° 58° 50°
420	42° 00'	136.419	88°	12°	215.572	320.479	83.579	57°	22° 42° 49° 58° 50°
430	43° 00'	133.246	92°	27°	231.276	318.053	103.726	57°	22° 42° 49° 58° 50°
440	44° 00'	130.218	96°	48°	247.789	314.46	130.99	57°	22° 42° 49° 58° 50°
450	45° 00'	127.324	100°	15°	265.071	309.482	161.94	57°	22° 42° 49° 58° 50°
460	46° 00'	124.556	105°	48°	283.38	309.59	193.43	57°	22° 42° 49° 58° 50°
470	47° 00'	121.906	110°	27°	302.007	295.354	227.294	57°	22° 42° 49° 58° 50°
480	48° 00'	119.366	115°	12°	321.698	285.967	263874	57°	22° 42° 49° 58° 50°

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Table of Spirals. Rate of Increase / in 20'

\angle	Deg	Radius	$\Delta.$	X.	Y.	X.	Y.	$\Delta.$	D.
0	0° 00'	Infinity	0° 00'	0.00	0.00	0.00	0.00	0° 00'	00'
10	0° 30'	1/145 9.16	0° 01'	.001	.000	.000	.000	0° 00'	00'
20	1° 00'	572 9.58	0° 06'	.006	.001	.000	.000	0° 00'	00'
30	1° 30'	381 9.72	0° 13'	.037	.000	.000	.000	0° 00'	00'
40	2° 00'	2864.79	0° 24'	.043	.000	.000	.000	0° 00'	00'
50	2° 30'	2291.83	0° 37'	.182	.999	.046	.000	1/2°	30°
60	3° 00'	1909.86	0° 54'	.314	.998	.078	.000	17°	57°
70	3° 30'	1635.02	1° 13'	.499	.997	.125	.000	24°	29°
80	4° 00'	1432.39	1° 36'	.745	.997	.187	.999	32°	0°
90	4° 30'	1273.24	2° 01'	1.060	.989	.265	.998	40°	29°
100	5° 00'	1145.91	2° 30'	1.457	.981	.363	.997	49°	40°
110	5° 30'	1041.74	3° 01'	1.936	.969	.484	.995	0°	30°
120	6° 00'	9579.93	3° 36'	2.573	.953	.629	.922	1/2°	0°
130	6° 30'	881.47	4° 13'	3.195	.939	.799	.988	1°	24°
140	7° 00'	818.51	4° 54'	3.991	.917	1.000	.982	1°	30°
150	7° 30'	763.94	5° 37'	4.909	.856	1.230	.977	1°	52°
160	8° 00'	716.19	6° 24'	5.957	.800	1.494	.968	2°	36°
170	8° 30'	674.07	7° 13'	7.146	.730	1.794	.953	2°	24°
180	9° 00'	636.62	8° 06'	8.482	.640	2.131	.937	2°	38°
190	9° 30'	603.11	9° 01'	9.976	.527	2.509	.920	3°	46°
200	10° 00'	572.95	10° 00'	11.636	.416	2.932	.899	3°	23°
210	10° 30'	545.67	11° 01'	13.470	.329	3.399	.877	3°	55°
220	11° 00'	520.87	12° 06'	15.487	.249	3.915	.835	4°	23°
230	11° 30'	498.22	13° 38'	17.696	.167	4.485	.791	4°	34°
240	12° 00'	477.46	14° 24'	20.000	.084	5.112	.745	4°	35°

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Table of Spirals. Rate of Increase 1/m 20'

L	Deg.	Radius.	Δ .	X	Y	X_0	Y_0	D	$D.$
250	12° 30'	458.37	15° 37'	22.726	248.141	5.799	24.684	5° 13'	54° 25° 16' 14"
260	13° 00'	440.73	16° 57'	25.563	257.719	6.549	129.598	5° 39'	52° 23° 53° 17"
270	13° 30'	424.42	18° 30'	28.620	267.268	7.365	134.531	6° 06'	50° 23° 03' 12"
280	14° 00'	409.25	19° 36'	31.928	276.723	8.250	139.439	6° 34'	54° 22° 35' 38"
290	14° 30'	395.15	21° 01'	35.473	286.095	9.207	149.325	7° 03'	54° 22° 20' 12"
300	15° 00'	381.98	22° 30'	39.270	295.373	10.235	149.196	7° 34'	24° 22° 22° 46"
310	15° 30'	369.65	24° 01'	43.330	304.536	11.339	154.038	8° 05'	38° 22° 30' 32"
320	16° 00'	358.10	25° 36'	47.659	313.579	12.539	158.849	8° 39'	31° 22° 45' 42"
330	16° 30'	347.25	27° 13'	52.269	322.549	13.799	163.698	9° 12'	18° 23° 03' 32"
340	17° 00'	337.04	28° 54'	57.166	331.357	15.193	168.473	9° 47'	19° 23° 24' 28"
350	17° 30'	327.42	30° 37'	62.360	340.000	16.691	173.207	10° 23'	36° 23° 50' 32"
360	18° 00'	318.31	32° 24'	67.859	348.487	18.307	177.928	11° 01'	08° 24° 31' 43"
370	18° 30'	309.71	34° 13'	73.672	356.793	20.041	182.598	11° 40'	00° 25° 08' 01"
380	19° 00'	301.51	36° 06'	79.809	364.914	21.916	187.265	12° 20'	21° 25° 49' 27"
390	19° 30'	293.82	38° 01'	86.277	372.821	23.911	191.866	13° 01'	49° 26° 31' 46"
400	20° 00'	286.48	40° 00'	93.085	380.502	26.061	196.356	13° 44'	50° 27° 16' 42"
410	20° 30'	279.49	42° 01'	100.242	387.941	28.372	200.835	14° 29'	18° 28° 04' 22"
420	21° 00'	272.84	44° 00'	107.757	395.116	30.860	205.243	15° 15'	11° 28° 54' 51"
430	21° 30'	266.49	46° 13'	115.639	402.020	33.514	209.598	16° 02'	55° 29° 47' 12"
440	22° 00'	260.44	48° 24'	123.876	408.600	36.369	213.843	16° 52'	01° 30° 42° 07"
450	22° 30'	254.62	50° 37'	132.537	414.866	39.4	22.031	17° 42'	19° 31° 40° 01"
460	23° 00'	249.11	52° 54'	141.570	420.785	42.925	222.099	18° 35'	32° 40° 06' 06"
470	23° 30'	243.81	55° 30'	151.005	426.333	45.0	226.068	19° 30'	16° 33° 42° 37%"
480	24° 00'	238.73	57° 36'	160.851	431.431	47.229	229.919	20° 26'	46° 34° 47° 39%"

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Table of Spirals. Rate of Increase $1^{\circ} m 30'$

\angle	Deg.	Radius	Δ	X	Y	X_0	Y ₀	D.	D.
0	0° 00'	000000.00	0° 00'	00	000	0.00	0.00	0.00	0° 00' 00"
10	0° 20'	17188.75	0° 01'	00	002	10.00	0.00	5.00	0° 00' 45"
20	0° 40'	8594.32	0° 04'	00	008	20.00	0.00	10.00	0° 01' 30"
30	1° 00'	5729.55	0° 09'	00	026	30.00	0.01	15.00	0° 03' 00"
40	1° 20'	4297.5	0° 16'	00	062	40.00	0.02	20.00	0° 04' 40"
50	1° 40'	3477.75	0° 25'	00	120	50.00	0.03	25.00	0° 08' 00"
60	2° 00'	2864.77	0° 36'	00	210	60.00	0.05	30.00	0° 12' 00"
70	2° 20'	2455.52	0° 49'	00	330	70.00	0.08	35.00	0° 16' 20"
80	2° 40'	2148.57	1° 04'	00	500	80.00	0.12	40.00	0° 21' 30"
90	3° 00'	1909.85	1° 21'	00	710	89.99	0.18	45.00	0° 27' 00"
100	3° 20'	1718.86	1° 40'	00	970	99.99	0.24	50.00	0° 33' 20"
110	3° 40'	1562.60	2° 01'	00	1290	109.99	0.32	55.00	0° 40' 20"
120	4° 00'	1432.39	2° 24'	00	1670	119.98	0.42	60.00	0° 47' 50"
130	4° 20'	1322.10	2° 49'	00"	2130	129.97	0.53	65.00	0° 56' 20"
140	4° 40'	1227.76	3° 16'	00"	2650	139.95	0.66	70.00	1° 05' 10"
150	5° 00'	1145.91	3° 45'	00"	3270	146.93	0.82	75.00	1° 15' 00"
160	5° 20'	1074.28	4° 16'	00	3980	159.92	0.99	79.99	1° 25' 30"
170	5° 40'	1011.09	4° 49'	00"	4760	169.88	1.19	84.99	1° 36' 20"
180	6° 00'	954.93	5° 24'	00	5650	179.84	1.41	89.98	1° 48' 00"
190	6° 20'	904.66	6° 01'	00	6650	189.80	1.66	94.97	2° 00' 20"
200	6° 40'	8594.3	6° 40'	00"	7750	199.72	1.94	99.95	2° 13' 20" 15"
210	7° 00'	818.57	7° 21'	00"	8970	209.64	2.24	104.93	2° 27' 00" 42"
220	7° 20'	781.30	8° 04'	00"	10320	219.54	2.57	109.92	2° 41' 30" 56"
230	7° 40'	747.30	8° 49'	00"	11780	229.44	2.95	14.90	2° 56' 20" 26"
240	8° 00'	716.20	9° 36'	00"	13464	239.32	3.43	119.88	3° 12' 00" 48"

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Table of Spirals. Rates of Increase, $^{\circ}/m 30'$

\angle	Deg	Radius	Δ	X	Y	X_o	Y_o	D	D_o
2 $^{\circ}$ 50'	8 $^{\circ}$ 20'	687.77	1 $^{\circ} 0'$ 25 $''$.00	15.150	249.17	3.82	124.81	3 $^{\circ}$ 28 $''$ 45 $''$
2 $^{\circ}$ 60'	8 $^{\circ}$ 40'	661.10	1 $^{\circ} 1'$ 16 $''$.00	17.041	258.99	4.39	129.83	3 $^{\circ}$ 45 $''$ 50 $''$
2 $^{\circ}$ 70'	9 $^{\circ}$ 00'	636.62	1 $^{\circ} 2'$ 09 $''$.00	19.085	268.79	4.83	134.80	4 $^{\circ}$ 03 $''$ 40 $''$
2 $^{\circ}$ 80'	9 $^{\circ}$ 20'	613.85	1 $^{\circ} 3'$ 04 $''$.00	21.285	278.54	5.39	139.76	4 $^{\circ}$ 22 $''$ 10 $''$
2 $^{\circ}$ 90'	9 $^{\circ}$ 40'	592.71	1 $^{\circ} 4'$ 01 $''$.00	23.648	288.26	6.00	144.70	4 $^{\circ}$ 41 $''$ 25 $''$
3 $^{\circ}$ 00'	10 $^{\circ}$ 00'	572.95	1 $^{\circ} 5'$ 00 $''$.00	26.179	297.95	6.66	149.66	5 $^{\circ}$ 01 $''$ 15 $''$
3 $^{\circ}$ 10'	10 $^{\circ}$ 20'	554.47	1 $^{\circ} 6'$ 01 $''$.00	28.885	307.57	7.36	154.58	5 $^{\circ}$ 21 $''$ 50 $''$
3 $^{\circ}$ 20'	10 $^{\circ}$ 40'	537.14	1 $^{\circ} 7'$ 04 $''$.00	31.771	317.17	8.12	159.53	5 $^{\circ}$ 43 $''$ 15 $''$
3 $^{\circ}$ 30'	11 $^{\circ}$ 00'	520.87	1 $^{\circ} 8'$ 09 $''$.00	34.844	326.69	8.93	164.43	6 $^{\circ}$ 05 $''$ 05 $''$
3 $^{\circ}$ 40'	11 $^{\circ}$ 20'	505.55	1 $^{\circ} 9'$ 16 $''$.00	38.109	336.16	9.79	169.34	6 $^{\circ}$ 28 $''$ 28 $''$
3 $^{\circ}$ 50'	11 $^{\circ}$ 40'	491.11	2 $^{\circ} 0'$ 25 $''$.00	41.571	345.56	10.71	174.24	6 $^{\circ}$ 51 $''$ 35 $''$
3 $^{\circ}$ 60'	12 $^{\circ}$ 00'	477.46	2 $^{\circ} 1'$ 36 $''$.00	45.238	354.88	11.70	179.72	7 $^{\circ}$ 15 $''$ 50 $''$
3 $^{\circ}$ 70'	12 $^{\circ}$ 20'	464.56	2 $^{\circ} 2'$ 49 $''$.00	49.113	364.13	12.76	183.78	7 $^{\circ}$ 40 $''$ 50 $''$
3 $^{\circ}$ 80'	12 $^{\circ}$ 40'	452.33	2 $^{\circ} 4'$ 04 $''$.00	53.203	373.30	13.80	188.84	8 $^{\circ}$ 06 $''$ 40 $''$
3 $^{\circ}$ 90'	13 $^{\circ}$ 00'	440.73	2 $^{\circ} 5'$ 21 $''$.00	57.516	382.37	15.07	193.67	8 $^{\circ}$ 32 $''$ 35 $''$
4 $^{\circ}$ 00'	13 $^{\circ}$ 20'	429.71	2 $^{\circ} 6'$ 41 $''$.00	62.054	391.34	16.35	198.49	9 $^{\circ}$ 00 $''$ 35 $''$
4 $^{\circ}$ 10'	13 $^{\circ}$ 40'	419.23	2 $^{\circ} 8'$ 01 $''$.00	66.805	400.20	17.68	203.28	9 $^{\circ}$ 28 $''$ 35 $''$
4 $^{\circ}$ 20'	14 $^{\circ}$ 00'	409.25	2 $^{\circ} 9'$ 24 $''$.00	71.848	408.94	19.14	208.04	9 $^{\circ}$ 57 $''$ 50 $''$
4 $^{\circ}$ 30'	14 $^{\circ}$ 20'	399.73	3 $^{\circ} 0'$ 49 $''$.00	77.090	417.56	20.65	212.78	10 $^{\circ}$ 27 $''$ 35 $''$
4 $^{\circ}$ 40'	14 $^{\circ}$ 40'	390.65	3 $^{\circ} 2'$ 16 $''$.00	82.594	426.04	22.27	217.49	10 $^{\circ}$ 58 $''$ 15 $''$
4 $^{\circ}$ 50'	15 $^{\circ}$ 00'	381.98	3 $^{\circ} 3'$ 45 $''$.00	88.355	434.39	23.98	222.17	11 $^{\circ}$ 29 $''$ 50 $''$
4 $^{\circ}$ 60'	15 $^{\circ}$ 20'	373.66	3 $^{\circ} 5'$ 16 $''$.00	94.376	442.57	25.80	226.83	12 $^{\circ}$ 02 $''$ 15 $''$
4 $^{\circ}$ 70'	15 $^{\circ}$ 40'	365.71	3 $^{\circ} 6'$ 49 $''$.00	100.667	450.59	27.73	231.43	12 $^{\circ}$ 35 $''$ 21 $''$
4 $^{\circ}$ 80'	16 $^{\circ}$ 00'	358.10	3 $^{\circ} 8'$ 24 $''$.00	107.230	458.44	29.67	236.01	13 $^{\circ}$ 09 $''$ 5d

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Table of Spins/s. Rate of Increase /° in 40'.

\angle	Deg	Radius	$\Delta.$	X	Y	X.	Y.	D.	D.
0	0° 00'	infinity	0° 00'	0.0	0.00	0.00	0.00	0° 00'	0° 00'
10	0° 15'	22918.31	0° 00'	45° 00'	10.00	0.00	5.00	0° 00'	22° 00'
20	0° 30'	11459.15	0° 00'	03° 00'	20.00	0.00	10.00	0° 00'	30° 00'
30	0° 45'	7639.42	0° 00'	06° 45°	0.02	30.00	0.00	15.00	0° 02'
40	1° 00'	5729.58	0° 00'	12° 00'	0.05	40.00	0.01	20.00	0° 05'
50	1° 15'	4583.66	0° 00'	18° 45°	0.09	50.00	0.02	25.00	0° 06'
60	1° 30'	3910.72	0° 00'	27° 00'	0.15	60.00	0.04	30.00	0° 09'
70	1° 45'	3274.04	0° 00'	36° 45°	0.25	70.00	0.06	35.00	0° 12'
80	2° 00'	2864.79	0° 00'	48° 00'	0.37	80.00	0.09	40.00	0° 16'
90	2° 15'	2546.48	1° 00'	45° 00'	0.53	90.00	0.13	45.00	0° 20'
100	2° 30'	2291.83	1° 00'	15° 00'	0.73	100.00	0.18	50.00	0° 25'
110	2° 45'	2083.48	1° 00'	30° 45°	0.97	110.00	0.24	55.00	0° 30'
120	3° 00'	1909.86	1° 00'	48° 00'	1.25	119.99	0.31	60.00	0° 36'
130	3° 15'	1762.95	2° 00'	06° 45°	1.60	129.98	0.40	65.00	0° 42'
140	3° 30'	1637.02	2° 00'	27° 00'	1.99	139.97	0.49	70.00	0° 49° 00'
150	3° 45'	1527.88	2° 00'	48° 45°	2.45	149.96	0.61	75.00	0° 56° 15"
160	4° 00'	1432.39	3° 00'	12° 00'	2.98	159.95	0.74	80.00	0° 64° 00"
170	4° 15'	1348.14	3° 00'	36° 45°	3.57	169.93	0.89	85.00	0° 72° 15"
180	4° 30'	1273.24	4° 00'	03° 00'	4.24	179.91	1.06	90.00	0° 80° 00"
190	4° 45'	1206.23	4° 00'	45° 00'	4.99	189.89	1.24	94.99	0° 80° 15"
200	5° 00'	1145.91	5° 00'	00° 00"	5.82	199.84	1.45	99.98	0° 90° 25° 32"
210	5° 15'	1091.35	5° 00'	45° 00"	6.73	209.80	1.68	104.97	0° 50° 33° 43"
220	5° 30'	1041.74	6° 00'	03° 00"	7.74	219.75	1.93	109.96	0° 01° 21° 23° 59°
230	5° 45'	996.44	6° 00'	45° 00"	8.84	229.69	2.22	114.84	2° 12° 15° 16° 34° 49°
240	6° 00'	954.93	7° 00'	12° 00"	10.05	239.62	2.51	119.93	2° 24° 30° 40° 44° 00"

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Table of Spirals. Rate of Increase 1°, n 40.

<i>L</i>	Deg.	Radius	<i>A.</i>	<i>X.</i>	<i>Y.</i>	<i>X.</i>	<i>Y.</i>	<i>D.</i>
250	6° 15'	9/6.73	7° 48' 45"	11.37	249.53	2.86	124.92	2° 36' 30'
260	6° 30'	881.47	8° 27' 00"	12.77	259.44	3.19	129.91	2° 49' 20'
270	6° 45'	848.83	9° 6' 45"	14.30	269.33	3.57	134.89	2° 02' 20'
280	7° 00'	818.51	9° 48' 00"	15.95	279.19	4.00	139.87	3° 06' 10'
290	7° 15'	790.29	10° 30' 45"	17.72	289.03	4.45	144.84	3° 16' 10'
300	7° 30'	763.94	11° 15' 00"	19.60	298.85	4.93	149.81	3° 45' 00"
310	7° 45'	739.30	12° 00' 45"	21.63	308.65	5.44	154.78	4° 00' 20"
320	8° 00'	716.20	12° 48' 00"	23.79	318.40	5.99	159.74	4° 16' 20"
330	8° 15'	694.49	13° 36' 45"	26.09	328.13	6.58	164.70	4° 32' 30"
340	8° 30'	674.07	14° 27' 00"	28.53	337.84	7.21	169.65	4° 49' 40"
350	8° 45'	654.81	15° 18' 45"	31.12	347.50	7.88	174.60	5° 07' 00"
360	9° 00'	636.62	16° 12' 00"	33.89	357.12	8.61	179.54	5° 25' 11"
370	9° 15'	619.41	17° 06' 45"	36.82	366.70	9.39	184.48	5° 44' 02"
380	9° 30'	603.11	18° 03' 00"	39.89	376.23	10.21	189.36	6° 03' 09"
390	9° 45'	587.64	19° 00' 45"	43.12	385.70	11.06	194.27	6° 22' 43"
400	10° 00'	572.95	20° 00' 00"	46.52	395.12	11.97	199.16	6° 42' 43"
410	10° 15'	558.98	21° 00' 45"	50.11	404.48	12.94	204.05	7° 03' 44"
420	10° 30'	545.67	22° 03' 00"	53.86	413.79	13.45	208.93	7° 24' 58"
430	10° 45'	532.98	23° 06' 45"	57.80	422.99	15.02	213.79	7° 46' 50"
440	11° 00'	520.87	24° 00' 00"	61.93	432.15	16.16	218.65	8° 09' 16"
450	11° 15'	509.29	25° 18' 45"	66.25	441.21	17.35	223.50	8° 32' 20"
460	11° 30'	498.22	26° 27' 00"	70.76	450.20	18.61	228.29	8° 55' 58"
470	11° 45'	487.62	27° 36' 45"	75.48	459.08	19.94	233.08	9° 20' 12"
480	12° 00'	477.46	28° 48' 00"	80.40	467.77	21.34	237.75	9° 45' 10"

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Table of Spirals. Rate of Increase in 5°.

<i>L</i>	Deg	Radius.	Δ_i	X	Y	X_o	Y_o	D	<i>D.</i>
0	0°	Infinity	0°	00' 00"	.000	00.000	00' 00"	00'	00"
10	12°	28647.90	0°	00' 36"	.001	10.000	00' 00"	22"	
20	24°	4323.95	0°	02' 24"	.004	20.000	00' 00"	42"	
30	36°	9549.30	0°	05' 24"	.016	30.000	00' 00"	50"	
40	48°	7161.97	0°	09' 36"	.037	40.000	00' 00"	63' 10"	
50	60°	5729.58	0°	15' 00"	.073	50.000	00' 00"	05' 01"	
60	72°	4774.65	0°	21' 36"	.126	60.000	00' 00"	07' 13"	
70	84°	4092.55	0°	29' 24"	.200	69.999	00' 00"	09' 48"	
80	96°	3580.98	0°	38' 24"	.298	79.998	00' 00"	12' 46"	
90	108°	3183.10	0°	48' 36"	.424	89.996	00' 00"	16' 12"	
100	120°	2864.79	1°	00' 00"	.582	99.994	00' 00"	20' 12"	
110	132°	2604.35	1°	12' 36"	.775	109.990	00' 00"	24' 13"	
120	144°	2387.32	1°	26' 24"	1.001	119.985	00' 00"	28' 39"	
130	156°	2203.68	1°	41' 24"	1.278	129.977	00' 00"	33' 46"	
140	168°	2046.27	1°	57' 36"	1.597	139.967	00' 00"	39' 13"	
150	180°	1909.86	2°	15' 00"	1.964	149.959	00' 00"	45' 02"	
160	192°	1794.49	2°	33' 36"	2.384	159.936	00' 00"	51' 14"	
170	204°	1685.17	2°	53' 24"	2.859	169.911	00' 00"	57' 49"	
180	216°	1591.55	3°	14' 24"	3.394	179.885	00' 00"	03' 54"	
190	228°	1507.75	3°	36' 36"	3.941	189.849	00' 00"	12' 15"	
200	240°	1432.39	4°	00' 00"	4.555	199.805	00' 00"	20' 04"	92° 23' 16"
210	252°	1364.18	4°	24' 36"	5.389	209.753	00' 00"	28' 17"	55' 21"
220	264°	1302.17	4°	50' 24"	6.196	219.686	00' 00"	36' 17"	28' 15"
230	276°	1245.56	5°	17' 24"	7.080	229.608	00' 00"	45' 17"	26' 54"
240	288°	1193.66	5°	45' 36"	8.044	239.515	00' 00"	55' 24"	24' 23"

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Table of Spirals. Rate of Increase 1° in 50'

L	Deg.	Radius	Δ	X	Y	X_0	Y_0	D	D_1
250	5° 00'	145.91	6° 15' 00"	9.092	249.405	2.281	1246.53	2° 05' 15"	10° 25' 39"
260	5° 12'	101.84	6° 45' 36"	10.228	359.270	2.572	129.572	1.5° 34"	9° 47' 27"
270	5° 24'	106.03	7° 17' 24"	11.454	269.125	2.878	134.491	2° 26' 13"	9° 24' 30"
280	5° 36'	1023.13	7° 50' 24"	12.724	278.950	3.210	139.388	2° 32' 19"	9° 11' 27"
290	5° 48'	987.86	8° 24' 36"	14.192	288.750	3.519	144.270	2° 48' 48"	9° 05' 43"
300	6° 00'	954.83	9° 00' 00"	15.712	298.519	3.955	149.35	3° 00' 45"	9° 04' 14"
310	6° 12'	924.12	9° 36' 36"	17.336	308.251	4.368	153.977	3° 13' 02"	9° 06' 24"
320	6° 24'	895.24	10° 19' 24"	19.068	317.955	4.810	158.806	3° 25' 54"	9° 11' 25"
330	6° 36'	868.12	10° 53' 24"	20.912	327.615	5.279	163.605	3° 39' 12"	9° 18' 48"
340	6° 48'	842.58	11° 33' 36"	22.872	337.231	5.781	168.382	3° 52' 57"	9° 28' 09"
350	7° 00'	818.51	12° 15' 00"	24.950	346.799	6.313	173.130	4° 06' 47"	9° 38' 59"
360	7° 12'	795.63	12° 57' 36"	27.150	356.315	6.882	177.876	4° 21' 26"	9° 51' 02"
370	7° 24'	774.26	13° 41' 24"	29.476	365.722	7.480	182.529	4° 36' 26"	10° 04' 55"
380	7° 36'	753.89	14° 26' 24"	31.931	375.170	8.114	187.175	4° 51' 54"	10° 19' 50"
390	7° 48'	734.56	15° 12' 36"	34.519	384.570	8.788	191.702	5° 07' 47"	10° 35' 50"
400	8° 00'	716.19	16° 00' 00"	37.243	393.932	9.499	195.350	5° 24' 11"	10° 52' 50"
410	8° 12'	698.72	16° 48' 36"	40.107	402.939	10.250	208.860	5° 41' 03"	11° 10' 45"
420	8° 24'	682.09	17° 38' 24"	43.113	412.034	11.041	205.335	5° 58' 23"	11° 29' 35"
430	8° 36'	666.22	18° 29' 24"	46.266	421.040	11.876	209.754	6° 16' 14"	11° 49' 19"
440	8° 48'	651.09	19° 21' 36"	49.570	429.948	12.760	214.107	6° 34' 36"	12° 09' 35"
450	9° 00'	636.62	20° 15' 00"	53.027	438.753	13.678	219.411	6° 53' 28"	12° 31' 19"
460	9° 12'	622.78	21° 09' 36"	56.641	447.447	14.651	222.636	7° 12' 33"	12° 53' 35"
470	9° 24'	609.53	22° 05' 24"	60.816	456.022	15.673	226.778	7° 32' 48"	13° 16' 42"
480	9° 36'	596.83	23° 02' 24"	64.355	464.470	16.667	230.886	7° 53' 08"	13° 40' 35"

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Table of Spirals. Rate of Increase 1° in 60°.

L	Deg	Radius	Δ	X	Y	X_0	Y_0	D	D	D
0	0°	Infinity.		0.000	0.000	0.00	0.00	00'	00'	00'
10	10°	34377.50		0.000	10.000	.000	5.00	20"	20"	20"
20	20°	17188.75		0.004	<0.000	.001	10.00	40"	40"	40"
30	30°	11457.20		-	30.000	.012	15.00	10"	10"	10"
40	40°	8594.52		0.031	40.000	.008	20.00	10"	10"	10"
50	50°	6875.46		0.061	50.000	.013	25.00	30"	30"	30"
60	60°	5727.55		0.105	60.000	.027	30.00	5"	5"	5"
70	70°	4911.17		0.166	70.000	.044	35.00	7.	10"	10"
80	80°	4297.15		0.248	79.999	.076	40.00	9.	30"	30"
90	90°	3819.70		0.324	89.999	.087	45.00	12'	10"	10"
100	100°	3437.75		0.405	99.999	.107	50.00	15'	15'	15'
110	110°	3125.21		0.485	109.999	.145	55.00	18'	30"	30"
120	120°	2864.77		0.56	109.999	.206	60.00	22'	10"	10"
130	130°	2644.41		0.645	119.999	.272	65.00	26	10"	10"
140	140°	2455.52		0.72	139.999	.323	70.00	30	30"	30"
150	150°	2291.82		0.80	1636 / 144.448	.410	75.00	35	10"	10"
160	160°	2146.57		0.88	1785 / 154.474	.503	80.00	40	10"	10"
170	170°	2022.19		0.96	2382 / 169.971	.592	85.00	45	30"	30"
180	180°	1904.85		1.04	330 / 179.962	.707	90.00	50	10"	10"
190	190°	1809.33		1.12	827 / 184.950	.828	95.00	57'	10"	10"
200	200°	1718.66		1.20	325 / 194.936	.973	100.00	60	30"	30"
210	210°	1637.01		1.28	878 / 209.918	1.18	105.00	65	10"	10"
220	220°	1556.70		1.36	490 / 219.896	1.287	109.99	70	24°	23°
230	230°	1475.00		1.44	562 / 229.870	1.475	109.99	75	14°	14°
240	240°	1403.39		1.52	30" / 5.899	1.674	30" / 1.44	80	46°	46°
240	240°	1400.00		1.60	0" / 239.849	1.674	0" / 1.44	80	11°	11°

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Table of Spirals. Rate of Increase in 60°.

L	Deg	Radius	Δ	X	Y	X_0	Y_0	D	D
2.50	4° 10'	375.09	5° 0'	7.580	249.804	1.941	124.97	1° 40'	39.00
2.60	4° 20'	322.10	5° 00'	8.521	259.758	2.135	129.97	1° 48'	30.00
2.70	4° 30'	273.20	5° 04'	9.542	269.710	2.412	134.97	1° 56'	29.00
2.80	4° 40'	227.76	6° 00'	10.642	279.653	2.674	139.96	2° 05'	47.00
2.90	4° 50'	185.40	7° 00'	11.824	269.586	2.971	144.95	2° 14'	40.00
3.00	5° 00'	145.91	7° 30'	13.090	294.570	3.290	149.92	2° 24'	30.00
3.10	5° 10'	108.95	8° 00'	14.443	309.421	3.635	154.91	2° 34'	30.00
3.20	5° 20'	1074.28	8° 32'	15.886	319.324	3.994	159.89	2° 44'	30.00
3.30	5° 30'	1041.73	9° 04'	17.451	329.211	4.403	164.88	2° 55'	10.00
3.40	5° 40'	1011.09	9° 38'	19.055	339.084	4.940	169.87	3° 06'	10.00
3.50	5° 50'	982.21	10° 12'	20.786	348.941	5.238	174.86	3° 17'	30.00
3.60	6° 00'	954.93	10° 48'	22.619	358.781	5.707	179.84	3° 28'	10.00
3.70	6° 10'	929.11	11° 24'	30	24.557	368.602	6.198	184.82	3° 40'
3.80	6° 20'	904.66	12° 02'	00	26.602	378.402	6.727	189.79	3° 52'
3.90	6° 30'	881.50	12° 40'	30	28.759	388.181	7.276	194.76	4° 05'
4.00	6° 40'	859.43	13° 20'	00	31.027	399.936	7.857	199.73	4° 18'
4.10	6° 50'	838.47	14° 00'	30	33.433	40.665	8.485	204.70	4° 30'
4.20	7° 00'	818.57	14° 42'	00	35.947	36.6	9.126	209.64	4° 55'
4.30	7° 10'	799.47	15° 24'	30	38.546	427.037	9.812	214.61	5° 09'
4.40	7° 20'	781.30	16° 08'	00	41.297	436.676	10.529	219.57	5° 23'
4.50	7° 30'	763.90	16° 52'	30	44.177	446.281	11.283	224.52	5° 38'
4.60	7° 40'	747.30	17° 38'	00	47.188	455.849	12.072	229.47	5° 53'
4.70	7° 50'	731.40	18° 24'	30	50.333	465.777	12.907	234.39	6° 08'
4.80	8° 00'	716.20	19° 12'	00	53.615	474.664	13.780	239.32	6° 24'

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Table of Sines - Rate of Increase 100.

L	Dy	Rat/S	S.	X.	Y.	Xo	Yo	D.	D.
0		1/7	00"	0	0	0	0	0	00"
10		583	6	222	0	10.00	0	5.0	00"
20		2291	3	30	.00291	20.00	.0007	10.0	00"
30		2230	1	22	.00920	30.00	.0022	15.0	30"
40		3046	1	00	.02327	40.00	.0061	20.0	08"
50		4273	1	6	.0221	40.00	.0115	25.0	00"
60		5236	1	9	.04545	50.00	.0197	30.0	30"
70		5236	1	3	.07054	60.00	.0311	35.0	43"
80		10000	1	8	.12472	70.00	.0464	40.0	59"
90		1371	1	4	.18617	80.00	.0660	45.0	07"
100		1379	1	0	.26507	90.00	.0866	50.0	10"
110		12236	1	7	.36361	99.9988	.0909	50.0	12"
120		3000	1	0	.48396	109.9981	.121	55.0	15"
130		3736	1	0	.62832	119.9970	.157	60.0	18"
140		4500	1	0	.79285	129.9956	.200	65.0	21"
150		5236	1	0	.997775	139.9936	.250	70.0	24"
160		6000	1	0	1.22778	149.9910	.307	75.0	28"
170		6730	1	0	1.48935	159.9875	.375	80.0	32"
180		1500	1	0	1.78642	169.9831	.447	85.0	36"
190		2230	1	5	2.1205	179.9775	.531	90.0	40"
200		3000	1	5	2.49390	189.9705	.624	95.0	45"
210		3730	1	2	3.13888	199.9619	.728	100.0	50"
220		4500	1	2	3.26339	209.9547	.842	105.0	55"
230		5230	1	8	3.87172	219.9587	.969	110.0	60"
240		6000	1	6	4.24144	229.9234	1.107	115.0	65"
			100	22	4.92654	239.9052	1.259	120.0	70"

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Table of Series Rate of Increase/Decrease

\angle	$\log \text{Series}$	X	Y	$X.$	$Y.$	$X.$	$Y.$	$X.$	$Y.$
370	4° 37' 30"	123	65.9743	247.253238	14.4221247.78	123	65.9743	247.253238	14.4221247.78
360	4° 36' 45"	123	65.9685	247.253238	14.4221247.78	123	65.9685	247.253238	14.4221247.78
350	4° 35' 00"	123	65.9627	247.253238	14.4221247.78	123	65.9627	247.253238	14.4221247.78
340	4° 34' 15"	123	65.9569	247.253238	14.4221247.78	123	65.9569	247.253238	14.4221247.78
330	4° 33' 30"	123	65.9511	247.253238	14.4221247.78	123	65.9511	247.253238	14.4221247.78
320	4° 32' 45"	123	65.9453	247.253238	14.4221247.78	123	65.9453	247.253238	14.4221247.78
310	4° 31' 00"	123	65.9395	247.253238	14.4221247.78	123	65.9395	247.253238	14.4221247.78
300	4° 30' 15"	123	65.9337	247.253238	14.4221247.78	123	65.9337	247.253238	14.4221247.78
290	4° 29' 30"	123	65.9279	247.253238	14.4221247.78	123	65.9279	247.253238	14.4221247.78
280	4° 28' 45"	123	65.9221	247.253238	14.4221247.78	123	65.9221	247.253238	14.4221247.78
270	4° 27' 00"	123	65.9163	247.253238	14.4221247.78	123	65.9163	247.253238	14.4221247.78
260	4° 26' 15"	123	65.9105	247.253238	14.4221247.78	123	65.9105	247.253238	14.4221247.78
250	4° 25' 30"	123	65.9047	247.253238	14.4221247.78	123	65.9047	247.253238	14.4221247.78
240	4° 24' 45"	123	65.8989	247.253238	14.4221247.78	123	65.8989	247.253238	14.4221247.78
230	4° 23' 00"	123	65.8931	247.253238	14.4221247.78	123	65.8931	247.253238	14.4221247.78
220	4° 22' 15"	123	65.8873	247.253238	14.4221247.78	123	65.8873	247.253238	14.4221247.78
210	4° 21' 30"	123	65.8815	247.253238	14.4221247.78	123	65.8815	247.253238	14.4221247.78
200	4° 20' 45"	123	65.8757	247.253238	14.4221247.78	123	65.8757	247.253238	14.4221247.78
190	4° 19' 00"	123	65.8699	247.253238	14.4221247.78	123	65.8699	247.253238	14.4221247.78
180	4° 18' 15"	123	65.8641	247.253238	14.4221247.78	123	65.8641	247.253238	14.4221247.78
170	4° 17' 30"	123	65.8583	247.253238	14.4221247.78	123	65.8583	247.253238	14.4221247.78
160	4° 16' 45"	123	65.8525	247.253238	14.4221247.78	123	65.8525	247.253238	14.4221247.78
150	4° 15' 00"	123	65.8467	247.253238	14.4221247.78	123	65.8467	247.253238	14.4221247.78
140	4° 14' 15"	123	65.8409	247.253238	14.4221247.78	123	65.8409	247.253238	14.4221247.78
130	4° 13' 30"	123	65.8351	247.253238	14.4221247.78	123	65.8351	247.253238	14.4221247.78
120	4° 12' 45"	123	65.8293	247.253238	14.4221247.78	123	65.8293	247.253238	14.4221247.78
110	4° 11' 00"	123	65.8235	247.253238	14.4221247.78	123	65.8235	247.253238	14.4221247.78
100	4° 10' 15"	123	65.8177	247.253238	14.4221247.78	123	65.8177	247.253238	14.4221247.78
90	4° 09' 30"	123	65.8119	247.253238	14.4221247.78	123	65.8119	247.253238	14.4221247.78
80	4° 08' 45"	123	65.8061	247.253238	14.4221247.78	123	65.8061	247.253238	14.4221247.78
70	4° 07' 00"	123	65.7993	247.253238	14.4221247.78	123	65.7993	247.253238	14.4221247.78
60	4° 06' 15"	123	65.7935	247.253238	14.4221247.78	123	65.7935	247.253238	14.4221247.78
50	4° 05' 30"	123	65.7877	247.253238	14.4221247.78	123	65.7877	247.253238	14.4221247.78
40	4° 04' 45"	123	65.7819	247.253238	14.4221247.78	123	65.7819	247.253238	14.4221247.78
30	4° 03' 00"	123	65.7761	247.253238	14.4221247.78	123	65.7761	247.253238	14.4221247.78
20	4° 02' 15"	123	65.7703	247.253238	14.4221247.78	123	65.7703	247.253238	14.4221247.78
10	4° 01' 30"	123	65.7645	247.253238	14.4221247.78	123	65.7645	247.253238	14.4221247.78
0	4° 00' 45"	123	65.7587	247.253238	14.4221247.78	123	65.7587	247.253238	14.4221247.78

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Table of Spirals. Rate of Increase / ° in 100'

L	Deg	Radius	Δ	X	Y	X.	Y.	D.
0	0°	Infinity.	0° 00' 00"	.000	0.000	.000	0.000	0° 00"
10	06	5729.580	0° 00' 18"	.000	10.000	.000	5.000	0° 00"
20	12	28647.90	0° 01' 12"	.002	20.000	.000	10.000	0° 15"
30	18	19098.60	0° 02' 42"	.008	30.000	.002	15.000	0° 00"
40	24	14323.95	0° 04' 48"	.019	40.000	.005	20.000	0° 40"
50	30	1459.16	0° 07' 30"	.036	50.000	.010	25.000	0° 02' 30"
60	36	9549.30	0° 10' 48"	.063	60.000	.017	30.000	0° 03' 35"
70	42	8185.11	0° 14' 42"	.100	70.000	.024	35.000	0° 04' 55"
80	48	7161.97	0° 19' 12"	.149	80.000	.037	40.000	0° 06' 25"
90	54	6366.20	0° 24' 18"	.212	90.000	.052	45.000	0° 08' 05"
100	60	5729.58	0° 30' 00"	.290	100.000	.072	50.000	0° 10' 00"
110	66	5208.70	0° 36' 18"	.387	110.000	.096	55.000	0° 12' 05"
120	72	4774.65	0° 43' 12"	.503	120.000	.128	60.000	0° 14' 25"
130	78	4407.37	0° 50' 42"	.640	129.999	.162	65.000	0° 16' 55"
140	84	4092.55	0° 58' 40"	.793	139.997	.199	70.000	0° 19' 35"
150	90	3819.72	1° 07' 30"	.982	149.995	.246	75.000	0° 22' 30"
160	96	3580.98	1° 16' 48"	1.191	159.992	.299	80.000	0° 25' 35"
170	102	3370.34	1° 26' 42"	1.429	169.989	.357	85.000	0° 29' 00"
180	108	3183.10	1° 37' 12"	1.696	179.986	.424	90.000	0° 32' 25"
190	114	3015.51	1° 48' 18"	1.995	189.981	.499	95.000	0° 36' 05"
200	120	2864.79	2° 00' 00"	2.327	199.974	.582	100.000	0° 40' 00"
210	126	2728.37	2° 12' 18"	2.693	209.969	.673	105.000	0° 44' 05"
220	132	2604.35	2° 25' 12"	3.097	219.961	.774	110.999	0° 48' 25"
230	138	2491.12	2° 38' 42"	3.539	229.951	.886	116.999	0° 52' 50"
240	144	2387.32	2° 52' 48"	4.021	239.939	1.004	122.999	0° 57' 35"

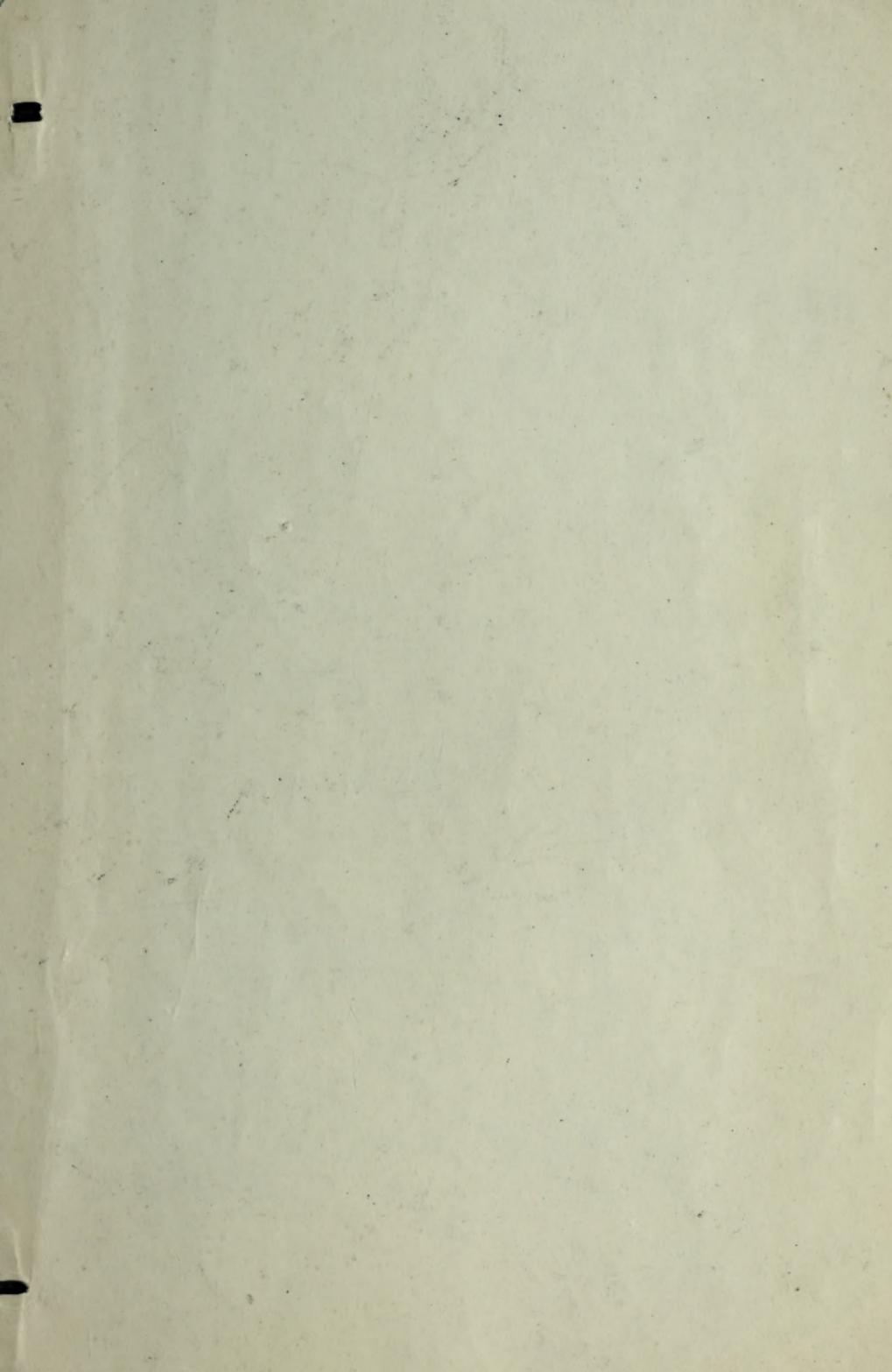
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Table of Spirals. Rate of Increase / m/100'

\angle	Deg.	Radius	$\Delta.$	X	Y	X_0	Y_0	D	$D.$
250	2° 30'	2291.03	3° 07' 30"	4545	249.926	1.35	24.986	1° 02'	30° 5° 12'
260	2° 36'	2203.68	3° 22' 48"	5.1/2	259.909	1.277	29.997	1° 07'	35° 4° 52'
270	2° 42'	2122.07	3° 38' 42"	5.725	269.891	1.433	34.994	1° 12'	50° 4° 41'
280	2° 48'	2046.27	3° 55' 12"	6.385	279.869	1.598	39.980	1° 18'	35" 4° 34'
290	2° 54'	1975.72	4° 12' 18"	7.094	289.844	1.776	44.985	1° 24'	05" 4° 30'
300	3° 00'	1909.86	4° 30' 00"	7.853	299.815	1.965	49.969	1° 30'	00" 4° 29'
310	3° 06'	1848.25	4° 48' 08"	8.665	309.782	2.170	54.964	1° 36'	10" 4° 30'
320	3° 12'	1790.49	5° 07' 12"	9.531	319.745	2.387	59.957	1° 42'	40" 4° 32'
330	3° 18'	1736.23	5° 26' 42"	10.453	329.702	2.619	64.935	1° 48'	33" 4° 36'
340	3° 24'	1685.17	5° 46' 48"	11.432	339.654	2.863	69.944	1° 55'	40" 4° 40'
350	3° 30'	1637.02	6° 07' 30"	12.471	349.600	3.120	74.933	2° 02'	34" 4° 45'
360	3° 36'	1591.27	6° 28' 48"	13.577	359.540	3.406	79.924	2° 09'	40" 4° 51'
370	3° 42'	1548.53	6° 50' 42"	14.733	369.472	3.707	84.915	2° 17'	20" 4° 58'
380	3° 48'	1507.79	7° 13' 12"	15.960	379.397	4.060	89.899	2° 24'	20" 5° 04'
390	3° 54'	1469.12	7° 36' 18"	17.253	389.313	4.380	94.886	2° 32'	15" 5° 12'
400	4° 00'	1432.39	8° 00' 00"	18.615	399.220	4.673	99.870	2° 40'	10" 5° 20'
410	4° 06'	1397.45	8° 24' 18"	20.046	409.118	5.037	204.854	2° 48'	20" 5° 28'
420	4° 12'	1364.18	8° 49' 12"	21.549	419.005	5.417	209.834	2° 56'	40" 5° 36'
430	4° 18'	1332.64	9° 14' 42"	23.125	428.881	5.816	214.802	3° 05'	10" 5° 46'
440	4° 24'	1302.18	9° 40' 48"	24.777	438.744	6.236	219.707	3° 13'	55" 5° 56'
450	4° 30'	1273.24	10° 07' 30"	26.505	448.605	6.676	224.756	3° 22'	50" 6° 03'
460	4° 36'	1245.56	10° 34' 48"	28.311	458.432	7.136	229.735	3° 32'	60" 6° 15'
470	4° 42'	1219.06	11° 02' 42"	30.198	468.254	7.617	234.704	3° 41'	25" 6° 25'
480	4° 48'	1193.66	11° 31' 12"	32.167	478.060	8.121	239.673	3° 51'	00" 6° 35'

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